Spectral inversion of source path and site terms using data from French Lesser Antilles (Guadeloupe and Martinique)

Stéphane Drouet and Marie-Paule Bouin

LGIT, CNRS, LCPC, Université Joseph Fourier, BP 53, 38041 Grenoble Cedex 9, France

IPGP, Observatoire Volcanologique et Sismologique de Guadeloupe, Le Houëlmont, 97113 Gourbeyre, France

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1 Data

On November 21, 2004, at 11:41 a magnitude M_w=6.3 (from the Harvard global Centroid Moment Tensor catalog, http://www.globalcmt.org/CMTsearch.htm earthquake struck the Guadeloupe (French Lesser Antilles) close to "Les Saintes" islands. It was followed by a large number of aftershocks in the following months.

The mainshock and its aftershocks were recorded by the French Accelerometric Network stations in Guadeloupe and Martinique, plus the accelerometric stations of the "Conseil Général de Martinique". The stations and data are managed by the "Observatoire Volcanologique et Sismologique de Guadeloupe" (OVSG) in Guadeloupe and by the "Observatoire Volcanologique et Sismologique de Martinique" (OVSM) in Martinique.

We selected all the events related to the "Les Saintes" seismic sequence (including one foreshock) recorded by at least 3 stations between November 2004 and May 2009 (see Table 4 in annex). The dataset includes 485 events recorded at 19 stations from Guadeloupe and 11 stations from Martinique (Fig. 1) which results in 2512 3-component records. The duration magnitudes (M_d), which are routinely estimated at the OVSG, range between 1.1 and 6.0 (for 68 events no magnitude was computed), and hypocentral distances range between 2.75 and 85.41 km for the stations in Guadeloupe and between 110.18 and 173.98 km for the stations in Martinique (Fig. 1). The list and location of the stations is indicated in Table 1.

Table 2: Moment magnitudes for the 6 largest events from the Harvard CMT catalog (http://www.globalcmt.org/CMTsearch.html).

Events date and time	M_w
2004.11.21-11.41.00	6.3
2004.11.21-13.30.00	5.3
2004.11.21-18.53.00	5.3
2004.11.27-23.44.00	4.9
2004.12.02-14.47.00	5.0
2005.02.14-18.05.00	5.8

To overcome this problem, we choose to impose the moment magnitude for the largest events. This will allow, for these events to adjust only the corner frequency, which is possible despite the limited frequency range. The Harvard CMT catalog contains 6 events relative to the "Les Saintes" seismic sequence (Table 2). These magnitudes will help to constrain the inversion (see the method section).

The upper frequency limit is 30 Hz, half the Nyquist frequency of the accelerometers. Fig. 2 shows the frequency range considered for each record; clearly data below 0.5 Hz are still usable for the largest event, while for the smallest event, the lowest usable frequency is around 1.2 Hz.

2 Method

In order to simultaneously determine source, path and site effects, far-field acceleration Fourier spectra of all the records are inverted using the methodology presented in Drouet et al. (2008) and Drouet et al. (2010). The time domain convolution of the three contributions becomes a simple multiplication in the spectral domain:

$$A_{ijk}(r_{ij}, f_k) = \Omega_i(f_k).D_{ij}(r_{ij}, f_k).S_j(f_k) \tag{1}$$

where r_{ij} is the hypocentral distance from earthquake i to station j and f_k the frequency.

The source is described using the usual Brune's source model (Brune, 1970, 1971):

$$\Omega_i(f_k) \sim \frac{(2\pi f_k)^2 M_{0_i}}{\left[1+\left(\frac{f_k}{f_{c_i}}\right)^2\right]} \tag{2}$$

where M_{0_i} is the seismic moment, and f_{c_i} the corner frequency of event i.

Attenuation involves anelastic decay and geometrical spreading:

$$D_{ij}(r_{ij}, f_k) = exp\left(-\frac{\pi r_{ij} f_k}{Q(f_k) v_S}\right) \times \frac{1}{r_{ij}^{\gamma}} \tag{3}$$

where v_S is the average S-wave velocity along the path and $Q(f_k) = Q_0 \times f_k^\alpha$ is the frequency-dependent quality factor. Note that the geometrical spreading may differ from the classical r_{ij}^{-1} form through the coefficient γ. We expect γ to be greater than 1, because downward reflections from layer interfaces (e.g., Frankel, 1991) and scattering (e.g., Gagnepain-Beyneix, 1987) can result in a geometrical loss of energy.

Finally, the base 10 logarithms of the Fourier spectra can be written as:

$$y_{ijk} = m_{0_i} - log_{10}\left[\left(\frac{(2\pi f_k)^2}{1+\left(\frac{f_k}{f_{c_i}}\right)^2}\right)\right] - \gamma log_{10}(r_{ij}) - \frac{\pi r_{ij} f_k^{1-\alpha}}{log_e(10) Q_0 v_S} + s_{jk} \quad (4)$$

where:

$$y_{ijk} = log_{10}\left[A_{ijk}(r_{ij}, f_k)\right] \quad (5)$$

$$m_{0_i} = log_{10}\left[M_{0_i} \times \frac{2R_{\theta\phi}}{4\pi\rho\beta^3}\right] \quad (6)$$

$$s_{jk} = log_{10}\left[S_j(f_k)\right] \quad (7)$$

with $R_{\theta\phi}$ the source radiation pattern, assumed to be constant ($R_{\theta\phi} = 0.55$ for S-waves, Boore and Boatwright, 1984), ρ the density, β the S-wave velocity of the medium at the source and v_S the S-wave velocity along the path (we assume $\beta = v_S = 3.5\, km.s^{-1}$ and $\rho = 2800\, kg.m^{-3}$). The factor 2 in Equation (6) accounts for the free surface reflection at the station assuming a quasi-vertical incidence. This is exact for SH and a reasonable approximation for quasi-vertical SV (Aki and Richards, 2002).

There is one degree of freedom left in equation 4 since all the seismic moments can be divided (or multiplied) by a constant term and the site terms can be multiplied (or divided) by the same constant without changing the equation (Andrews, 1986; Boatwright et al., 1991). The usual way to solve that problem is to impose that on average logarithms of site effects are null (no site effects on average). In this study we choose to impose the moment magnitudes for the 6 largest events, what we call the reference condition is then simply:

$$m_{0_{reference\ event}} = log_{10}\left[M_{0_{reference\ event}} \times \left(\frac{2R_{\theta\phi}}{4\pi\rho\beta^3}\right)\right] \pm 0.0001 \quad (8)$$

for all of the 6 events from Table 2.

The standard deviation we impose on the reference condition is small enough to ensure that the moment magnitudes for the large events will remain fixed during the inversion.

Table 3: Attenuation parameters determined in this study.

γ	Q_0^1	α^1	Q_0^2	α^2
Guadeloupe data only				
1.088 ± 0.003	244 ± 32	0.24 ± 0.02		
All data one Q model				
1.077 ± 0.001	345 ± 5	0.21 ± 0.01	–	–
All data two Q models				
1.058 ± 0.001	261 ± 12	0.16 ± 0.01	287 ± 5	0.33 ± 0.01

We use an iterative Gauss-Newton inversion scheme, based on the derivatives of y_{ijk} with respect to the parameters, to linearize the problem at each iteration and converge to the solution (Tarantola, 2004; Drouet et al., 2008).

3 Results

3.1 Attenuation

The data from Guadeloupe and Martinique are characterised by very different propagation paths. All the records from Guadeloupe have distance lower than 90 km and all the records from Martinique have distances greater than 110 km. Moreover, the paths are crossing different regions and different portions of the crust, the longest paths are going deeper in the crust. We performed tests in order to check the influence of the data used on the propagation path parameters determination. We first used only data from Guadeloupe and then data form Guadeloupe and Martinique under 2 hypothesis: 1) the quality factor is the same for all the data; 2) two quality factors are defined, one for the paths towards Guadeloupe and one for the paths towards Martinique. Table 3 shows that using all the data together with the same quality factor leads to slightly different values for Q_0 and α compared to the results using only data from Guadeloupe. It also shows that using two different quality factors leads to different α values for the two categories of paths. Due to the majority of data from Guadeloupe the attenuation parameters are not changing much by the inclusion of data from Martinique. However, as shown by the test with the two quality factors, the travel path towards Martinique are crossing less attenuating materials (high α means high quality factor) which is expected since those paths are going deeper into the crust. In the following the results from the 2 quality factors model are shown.

3.2 Residuals

After the inversion, we compute the residuals between the observed data and the synthetic model build with the inverted parameters. The residuals distribution is shown on Figure 3 for all the frequencies as well as for the low frequencies (1≤f≤2 Hz) and for the high frequencies (15≤f≤20 Hz). The parameters of the equivalent Gaussian distribution (mean: μ and standard deviation: σ) are also given and show a mean distribution around 0 with standard deviation almost independent of the frequency band around 0.149 to 0.164. The residuals are also plotted against hypocentral distance, duration magnitude and frequency in Figure 3. The distribution and the bin average residuals do not show any trend.

3.3 Source parameters

3.3.1 Magnitudes

Moment magnitudes are estimated from the inverted seismic moments using Hanks and Kanamori (1979) relationship:

$$M_w = \frac{log_{10}(M_0) - 9.1}{1.5} \tag{9}$$

They are compared with the routinely determined duration magnitude in Figure 4 which shows that M_d is systematically lower than M_w within the magnitude range analysed, the relationship being:

$$M_w = 0.504(\pm 0.031) + 1.010(\pm 0.011) \times M_d \tag{10}$$

3.4 Corner frequencies and Brune's stress drop

The Brune's source model predicts a relationship between seismic moment and corner frequency which involves the stress drop parameter: $\Delta\sigma$ (Brune, 1970, 1971):

$$M_0 = \frac{16}{7}\Delta\sigma\left(\frac{0.37v_S}{f_c}\right)^3 \tag{11}$$

Under the assumption of constant stress drop, there is a linear relationship between M_w (which is equivalent to $log_{10}(M_0)$) and corner frequency: $M_w \sim -(1/3)f_c$.

The corner frequencies are plotted against moment magnitudes in Figure 5, with the theoretically expected relationship for three different constant stress drop values (10^5, 10^6 and 10^7 Pa). The Figure shows that stress drops are almost all lying between 1 and 500 bars (1×10^5 and 500×10^5 Pa). The linear regression shows a clear increase of stress drop with magnitude with an average value around 10×10^5 Pa for M_w=2.5 and around 100×10^5 Pa for M_w=5.0. The extrapolation of this relationship toward larger magnitudes gives a relatively large stress drop for the main event (493 bars) compared to the value estimated from the inverted corner frequency (263 bars). We then fit the data with a segmented regression, for events with M_w<4.6 and for those with $M_w \geq$4.6, and with a polynomial function or order 3. Those two regressions show a flattening of the relationship between corner frequency and moment magnitude leading to more realistic stress drop values for the larger events. This would be in agreement with the transitional model of Walter et al. (2006). But the dependence of stress drop with magnitude is a highly debated issue (Ide and Beroza, 2001).

3.5 Radiated energy and apparent stress

The radiated energy for each event is also estimated through the integration of the squared velocity source spectra (Abercrombie, 1995; Mayeda and Walter, 1996). We first correct all the spectra of the same event for propagation and site effect in order to get source spectra which we also convert from acceleration to velocity:

$$V_{ijk} = \frac{A_{ijk}}{2\pi f_k} \times r_{ij}^{\gamma} \times exp\left(\frac{\pi r_{ij} f_k^{1-\alpha}}{Q_0 v_S}\right) \times S_j(f_k) \qquad (12)$$

The average velocity source spectra over all the stations that recorded each event is squared and integrated, and the energy carried by S-waves is estimated (Mayeda and Walter, 1996):

$$E = \frac{R_{\theta\phi}}{4\pi\rho v_S^5} \times \int_{f_1}^{f_2} V(f)^2 df \qquad (13)$$

In order to estimate the uncertainty associated with the energy estimation we have to compute the uncertainty linked to the average source velocity spectra. This is done by computing the uncertainty linked with the spectra correction following the error propagation model:

$$\sigma = \sqrt{\left(\sigma_\gamma \times \frac{\partial cor}{\partial \gamma}\right)^2 + \left(\sigma_{Q_0} \times \frac{\partial cor}{\partial Q_0}\right)^2 + \left(\sigma_\alpha \times \frac{\partial cor}{\partial \alpha}\right)^2 + \left(\sigma_{S_{ij}} \times \frac{\partial cor}{\partial S_{ij}}\right)^2} \qquad (14)$$

which is added to the standard deviation associated with averaging the source spectra obtained at different stations.

Finally, Ide and Beroza (2001) computed a correction function to account for missing high frequencies in the integration, which arise from recording limitations. Since most of the energy is released around the corner frequency, for small or large events the energy estimation might be biased by the frequency band limitation. The correction proposed by Ide and Beroza (2001) is applied to each energy estimation. In the following we will assume that the S-waves energy represents the radiated energy. P-waves also carry a part of the radiated energy but it is estimated to be less than 10 % of the total energy (Abercrombie, 1995; Mayeda and Walter, 1996).

Figure 5 shows that the relationship between seismic moment and radiated energy is linear throughout the seismic moment range. For the largest events with their corner frequency outside the frequency range, the energy estimation might be biased. This could explain the departure from the linear trend for the largest events. The correction from Ide and Beroza (2001) increases the energy by about 1%, which is apparently not enough to get a robust estimate.

We also compute apparent stress:

$$\sigma_a = \frac{2\mu E}{M_0} \tag{15}$$

with $\mu = 3.4 \times 10^{10}$ Pa. Since energy might be underestimated for the largest events, apparent stresses are probably also underestimated.

Figure 6 shows that the scaling observed between Brune's stress drop and seismic moment also exists between apparent stress and seismic moment. The Brune's stress drop values for the large events should not be biased although the frequency band is limited. However, one has to recall that the apparent stress drop values are underestimated for the largest events ($M_w \geq$4.6). Assuming a linear relationship between stress parameters and seismic moments, the observed rate of increase is higher than that determined by Mayeda and Walter (1996) and Bay et al. (2005) for the events with M_w <4.6. For the largest events, the linear fit leads to unrealistically high stress values. A segmented fit for events with M_w <4.6 and those with $M_w \geq$4.6, or an order 3 polynomial fit show a flattening of the relationship between stresses and seimic moments. This is consistent with the transitional model from Walter et al. (2006). Figure 6 also shows that Brune's stress drop and apparent stress are linearly correlated.

3.6 Site effects

Finally, we determine the site transfer functions for all the stations analysed in this study (Fig. 7). Due to the large amount of events recorded at each station (see Table 1), the uncertainty on the site amplification is very small. The station with the lowest number of records (MASP which recorded 2 events) presents also small uncertainty on the siite amplification. this is probably a consequence of the large number of events which allowed a robust estimation of source and propagation parameters. The stations with the flattest amplification functions and the lowest amplitudes are: ADEA, CGCA, IPTA, PIGA and MAMA. Those stations can be considered as rock reference stations.

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Table 1: Stations analysed in this study.

Name	Longitude	Latitude	Elevation (km)	Number of records
OVSG stations				
ABFA	61.74	16.01	0.018	189
ADEA	-61.09	16.30	0.009	8
BDRA	-61.46	16.49	0.081	10
CBEA	-61.56	16.05	0.029	38
CDSA	-61.70	15.98	0.426	16
GBGA	-61.32	15.88	0.009	284
IPTA	-61.53	16.23	0.026	247
JARA	-61.56	16.25	0.004	101
MESA	-61.46	16.33	0.023	76
MOLA	-61.36	16.31	0.025	89
PIGA	-61.77	16.15	0.008	225
PRFA	-61.72	16.09	0.006	279
SAGA	-61.36	16.25	0.025	14
STGA	-61.26	16.25	0.015	15
SNOA	61.71	16.33	0.020	148
TBPA	61.64	15.85	0.046	9
TDBA	61.64	15.85	0.276	841
TDHA	61.58	15.87	0.114	58
THMA	61.58	15.87	0.012	100
OVSM stations				
CGCA	61.17	14.70	0.015	11
CGDI	61.05	14.47	0.005	6
CGLR	-61.10	14.46	0.052	6
MADI	-61.04	14.49	0.090	9
MALA	-60.99	14.59	0.086	6
MAMA	-60.87	14.47	0.020	9
MAME	-61.12	14.61	0.140	9
MASM	-60.98	14.76	0.042	16
MASP	-61.17	14.74	0.010	2
MATR	-60.96	14.75	0.026	16
MAZM	-61.02	14.55	0.025	9

All the records were visually inspected to identify any problem (i.e. step function within the signal, spikes...) and to pick P- and S-waves arrival times. Figure 2 shows examples of data recorded at station GBGA for 3 different earthquakes with indication of the P- and S-waves arrival times. It shows that even very small earthquakes are well recorded. The Fourier spectra for noise are computed from the beginning of the recording to the P-wave arrival time. The Fourier spectra for S-waves are computed from the time window starting at the S-wave arrival time and ending were it includes 80% of the energy computed from the S-wave arrival time. The spectra are then smoothed using Konno and Ohmachi (1998) smoother and the two horizontal components are combined as: $H = sqrt(East - West \times North - South)$. Finally we keep only data with signal amplitude greater than 3 times the noise amplitude.

Since we are interested in very small earthquakes we limit the frequency range to 0.5 Hz were there is still significant energy in the signal for small earthquakes. This will, in turn, limit our ability to determine robust source parameters for the largest earthquakes, especially the main shock which corner frequency is likely smaller than 0.5 Hz. However since only three events have a duration magnitude greater than 4.5 (Fig. 1), the number of good quality data below 0.5 Hz is too small to be included. Indeed, in the inversion procedure, a large number of data at each frequency is required to simultaneously determine source, path and site parameters.